

SUDOKU PUZZLE SECRETS:

**Learn How to Solve Sudoku Puzzles
With Little Effort**

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INTRODUCTION

It seems that these days everyone is enjoying the game of Sudoku wherever they are. The Sudoku puzzle is ideal for whenever you have a few spare minutes and want to indulge in a little bit of thinking power. Sudoku, sometimes spelled "Su Doku", is a puzzle that originated in Japan. The puzzle is known as a "placement" puzzle. In the United States Sudoku is sometimes called the "Number Place" puzzle.

People of all ages and from all backgrounds are finding that Sudoku is a great way to keep their mind active and thinking. Puzzles range all the way from easy for the beginner to extremely difficult for the more advanced puzzler. Sudoku is easy to take with you wherever you go so that you can indulge in a little bit of number guessing whenever you have a few spare minutes.

Sudoku is easy to learn and understand. The main aim of Sudoku is to enter a number from one to nine into each cell on puzzle grid. The most frequent layout of a Sudoku puzzle is a 9 x 9 grid that is made of subgrids that are 3 x 3. Each of these subgrids is known as a "region". Depending on how easy or hard the puzzle is there will be various starting numbers in the cells. These are known as

the “givens”. Every row, column, and region of the Sudoku puzzle can contain only one instance of each number. You complete the puzzle when all of the cells have been filled in with corresponding numbers.

To complete the Sudoku puzzle requires a lot of patience as well as the ability to think logically. The basic layout of the Sudoku grid is much like a chess game or crossword puzzles. Sudoku is not just a mathematical or arithmetic type of puzzle. It works just as well if the numbers are substituted with letters or other symbols. However, numbers work best.

The bottom line is that Sudoku is a fascinating new puzzle game that has taken the world by surprise and storm. You can now find Sudoku in many national newspapers. The great thing about this puzzle is that the basic principle of solving it is really quite simple. All you need to do is fill in the grid in such a way that each row, column, and region contains the numbers one to nine.

CHAPTER I: HISTORY OF SUDUKO

You would imagine that with such a name this puzzle originated in Japan, but it has been around for many years in the United States and in the UK. However, the Japanese found an example under the title “Number Place” in an American magazine and translated it as something quite different: *su* meaning number and *doku* meaning single unit. It immediately caught on in Japan where number puzzles were much more prevalent than word puzzles. Crosswords don’t work very well in the Japanese language.

Sudoku was first published in the late 1970’s in North America in New York by the publisher “Dell Magazines”. Dell was known as a specialist when it came to puzzles of logic and ability. Dell published Sudoku as “Number Place” in its *Math Puzzles and Logic Problems* magazine.

It has not been recorded who designed the Americanized puzzle but suspicion falls on Walter Mackey who was one of Dell’s constructors of puzzles. In Japan, Sudoku was first introduced by Nikoli in 1984. The puzzle appeared in the Monthly Nikolist in April as “Suuji wa dokushin ni kagiru”. This can be translated to “the numbers must be there in only one instance”. In 1986 Nikoli introduced two

different versions of Sudoku as the popularity of the puzzle increased. No more than 30 “givens” were allowed that the grid became symmetrical. Sudoku is now published in many mainstream Japanese periodicals, including the *Asahi Shimbun*. The trademark name of Sudoku is still held by Nikoli while other publications in Japan use other names.

Sudoku quickly spread to the computer. In 1989 DigitHunt was created for the Commodore 64 by a company called Loadstar/Softdisk Publishing. This home computer version of Sudoku allowed people of all ages to enjoy the game in a computerized style.

Sudoku is now published in a variety of places including the New York Post and USA Today. The puzzle is also reprinted by Kappa in *GAMES* magazine. Many times you will find Sudoku included in puzzle anthologies which include *The Giant 1001 Puzzle Book*. In these books Sudoku is usually titled something like “Nine Numbers”. Surprisingly Dell, who invented the Americanized version of the puzzle, has failed to cash in on this big puzzle rage.

The Sudoku puzzle reached craze status in Japan in 2004 and the craze spread to the United States and the UK through pages of national newspapers. The Daily

Telegraph uses the name “Sudoku” but you may the puzzle called “su doku” in other places. However, there is no doubt that the word has been adopted into modern parlance, much like the word “crossword”.

CHAPTER 2: SUDOKO EXPLAINED

Sudoku can take on many different variant forms. The one main standard is that each of the numbers in a region needs to be unique. With so many different variations of the puzzle to choose from you will never run out of challenges. Start out slowly with the standard grid layout of 9 x 9 before you move on to one of the many variations that you can find. Your goal should be to solve the Sudoku puzzle with little “givens” as you can.

Variants

Sudoku is usually played as a 9 x 9 grid which contains 3 x 3 regions. Although this is the most common grid layout there are many variations which can be found. The following grid layouts are not uncommon and can add an even more challenging level to the puzzle:

- 4 x 4 grid with 2 x 2 regions

- 5 x 5 grid with pentomino regions (these puzzles are known as “Logi-5”)
- 6 x 6 grid with 2 x 3 regions (grid used in the World Puzzle Championship)
- 7 x 7 grid with six heptomino regions as well as a “disjoint” region.
- 9 x 9 grid that generally has nonomino regions

Larger grid puzzles are possible, such as the 16 x 16 grid layout published by Dell that is known as “Number Place Challenger”. As well, Nikoli in Japan as published a 25 x 25 grid. Yet another variant for the Sudoku puzzle is the for the numbers in the main diagonal areas to be completely unique.

Yet another variation of the Sudoku puzzle is “Gattai 5 Sudoku”. In this puzzle variation there are five 9 x 9 grids which overlap at the corner regions into the refined shape of a quincunx. In the New York Times this puzzles is known as “Samurai Su Doku”.

A popular Sudoku puzzle in 2005 was a three dimensional grid. This grid variation was invented by Dion Church and was first published in the Daily Telegraph. It became a fast hit among those puzzlers who wanted the ultimate in puzzle action.

There have been alphabetical variants of Sudoku where letters replace the numbers. This type of a puzzle is sometimes called "Godoku" and can be very difficult to solve if there are few "givens" available. Alphabetical Sudokus are also known as "Wordoku". The letters required for the puzzle will be given to you beneath the puzzle. After you have arranged the letters they will spell out a word that lines up between the top left bottom and left corners of the grid. This little twist adds a completely different dimension to the puzzle. There will be times when you can guess the word and this will aid you in solving the rest of the puzzle by filling in the rest of the regions.

Japanese Variants

There are many Japanese variations of Sudoku which appear in magazines all over the country. Each variation has its own challenges that attract different individuals. Following is a list of some of the Japanese variants which have been developed:

- Puzzles that are sequentially connected: Sequential puzzles have you solving several 9 x 9 grid Sudoku puzzles at one time. The first puzzle has enough "givens" in it so that it can be solved on its own.

After you have solved the first puzzle some of the numbers are moved from the first solved grid to the grid of the second 9 x 9 puzzle. You will have to work back and forth from one puzzle to the other to successfully solve these sequential Sudoku puzzles.

- Overlapping puzzles: One popular version of Sudoku are multiple overlapping puzzles. These large puzzles are made up mostly of 9 x 9 grids but often these grids deviate from the standard. It is not uncommon to have one puzzle made up of 20 to 50 standard 9 x 9 grids. Regions of each puzzle will overlap with one another. For instance, two 9 x 9 grids may have 9, 18, or 36 cells that are in common with each other. And other times there may no overlapping areas at all to connect with one another.
- Multiple cells: This variation of the Sudoku puzzle appears quite simple at first. Each cell in a 9 x 9 standard puzzle is part of four other puzzles rather than just the standard three parts – rows, columns, and regions. In this case numbers that are located within their region area can not match. This type of puzzle will usually be printed in color so that you can easily identify which area of the grid you are working on.
- “Digital Number Place”: In 2005 the World Puzzle Championships include a puzzle of this kind, calling it

“Digital Number Place”. Instead of being provided with a “given” most of the cells contained a partial given. A partial given is a segment of a number where some portions have been drawn as though they are part of a liquid crystal display.

With so many Sudoku variations to choose from you will be able to spend hours and hours facing the challenge of solving them.

Terminology and Rules

The Suduko puzzle is quite easy to solve, at least in the general concept. Your goal will be to fill in each of the empty cells with one number. Every row, column, and region will contain the numbers from one to nine exactly one time. This means that every number in the solution of the puzzle will occur only one time in three directions.

The reason that so many people are attracted to a Sudoku puzzle is that, even though the solving rules are simple, the reasoning behind the path to the correct solution can be very difficult. Most puzzles will be ranked according to how difficult they are. Still other puzzles will give you an estimated time of how long it should take you to solve the puzzle. In most cases, the more “givens”

there are, the easier the puzzle will be to solve. The bottom line on how easy it is to solve a Sudoku puzzle will depend on how easy it is to determine the logical order of all of the numbers.

Many teachers, no matter what age range they are teaching, recommend Sudoku as a great way to develop logical reasoning. The complexity of each puzzle can be adapted to fit any age.

CHAPTER 3: THE MATH BEHIND SUDOKU

The Sudoku puzzle is unlike most puzzles in that it is based on mathematical structure and requires some level of logic in order to be solved. The main basis behind solving Sudoku is called “NP-complete” because it is solved on $n^2 \times n^2$ grids of $n \times n$ cells. It is this concept that makes Sudoku so difficult to solve. When you put cells on grids and throw in a few “givens” it takes some determining finite power to solve the puzzle correctly.

Sudoku has what is known as a “game tree”. The game tree of this puzzle game is quite large and, when there is only one solution to be found, makes solving it fast an

unfeasible plan. There are, however, tips that you can use to solve Sudoku as fast as possible.

Perhaps an easy way of describing the solution of a Sudoku puzzle is to call it a "graph coloring problem". The basic goal of the puzzle is to build, in its standard form of 9×9 , a coloring grid. The entirety of the graph is composed of 81 vertices, with one vertex for every cell on the grid. Each of the vertices can be named with pairs that are ordered and where "x" and "y" are integers anywhere from one to nine. This means that two separate vertices are names and are connected by an edge if, and only if the edges match. The Sukoku puzzle is eventually solved by assigning an integer, from one to nine, to each of the vertices in a way where the vertices connected by an edge don't have the same integer assigned to them.

A Latin Square

The solution of the Sudoku grid is much like a Latin square. There are, however, less solution grids for Sudoku, than there are Latin squares. This is because Sudoku has the additional problem of multiple regions. Still, there are endless solution grids for the Sudoku puzzle. In 2005 Bertram Felgenhauer calculated the

number to be about 6,670,903,752,021,072,936,960. He arrived at this number using logical computations. The analysis of the number of solution grids was further simplified by Frazer Jarvis and Ed Russell. It has not yet been calculated how many solution grids there are for the 16 x 16 Sudoku puzzle.

There are some 9 x9 grids that can be recreated into other grids. This can be done by (1) rotating or reflecting the grid, (2) permuting some columns and rows, and (3) changing around the numbers. In 2005 Frazer Jarvis and Ed Russell calculated the number of different Sudoku grids that could be created and came up with a total of 5,472,730,538.

Unique Grids

In order to keep the Sudoku grid unique it's important not to provide too many "givens". The maximum number of "givens" that can be included in a puzzle before the grid solution is considered too unique is four less of a full grid. When there are two instances of two numbers which are each missing, and the cells which they are supposed to fill are each the corners of an orthogonal rectangle, there will only be two ways in which the numbers may be added together.

The opposite of this is just as true. The least number of “givens” that can be used before a solution is unique, or rather is a puzzle that can’t be solved, is 17. Some Japanese puzzle experts believe that this number is 18. Regardless how the least number of “givens” are rotated, the Sudoku puzzle will be unsolvable unless there are enough “givens” to make it symmetric.

Mathematically, the Sudoku puzzle is a work of art that has only one solution. This means that you may almost complete the puzzle only to find that it is one cell that turns out to be wrong. You will have no choice but to start over so that you can accurately place the numbers in the regions.

CHAPTER 4: CONSTRUCTION OF THE PUZZLE

The construction of the Sudoku puzzle is done in a variety of ways. In most cases, a “puzzle generator” will be used. It is generally thought that Dell uses a computer program to generate their puzzles. A Dell Sudoku puzzle will typically have over 30 “givens” which will be placed in random cells around the grid. Many of these “givens” will lead to the deduction of other obvious number placements. Dell, and other puzzle creators in North

America, seldom give any authoring credit to the Sudoku puzzles which they create.

Japanese creators of the Sudoku puzzle are always credited for their work. And most Japanese puzzles are created by hand. Another difference between the American Sudoku and the Japanese Sudoku is that Japanese puzzle creators generally place the "givens" in a symmetrical pattern. As a side note, the "givens" can be placed symmetrically on the grid by allotting a number to them and by deciding ahead of time where they will be placed.

When constructing Sudoku puzzles it is often possible to set each starting grid so that it has more than one solution and to set others so that there is no solution at all. These puzzles are not considered to be a true Sudoku puzzle. This is because when it comes to the general basis of Sudoku, a unique solution is always expected.

Creators of the Sudoku puzzle need to make sure that when they are constructing a grid that they understand where numbers can be logically placed. To overlook the final solution of the puzzle can lead to a grid that is unsolvable and which contradicts the basic premise of what Sudoku is all about.

When you are solving a Sudoku puzzle, and you place a digit randomly to the grid, you are one step closer to the solution but perhaps no closer to the right solution. You can randomly remove one digit and replace it with another but the logic behind the Sudoku puzzle is that you take the time to apply logic and mathematical reasoning.

CHAPTER 5: SOLUTION METHODS - SCANNING

Scanning is one way that you can solve a Sudoku puzzle. When you first look at that puzzle you should scan it at least once and again a few times while you are trying to arrive at the solution. Take some time to analyze the puzzle as you are working it since scanning can help you to quickly pick up on a working in one or two needed numbers.

There are two basic techniques when it comes to scanning: cross-hatching and counting. You can use both of these methods alternately.

You won't be able to scan the puzzle any further when you run out of numbers to put into cells. After this you will need to start working the puzzle from a logical stand point. Some people find that it helps to mark possible numbers in the cells. You can do this using either subscripts or dots:

- Subscript marking: Use subscript to mark possible number into the cells. The one disadvantage to this is that many puzzles, such as those found in newspapers, are often too small to allow you to write in the cells. Consider making a larger copy of the puzzle so that you can read it easier or use a pencil that is very sharp so that you can write fine lines.
- Dot marking: Dot marking involves using a pattern of dots. A dot in the top left will indicate a one and dot in the bottom right will indicate a 9. The advantage of using the dot notation is that you can easily use it on the original puzzle. You will have to make sure that you don't make a mistake with the dots or you will be led into confusion and it may not be easy to erase dots without creating more confusion.

Cross-hatching and Counting

Cross-hatching and counting are two natural methods you can use to help you solve your Sudoku puzzle.

Cross-hatching starts by scanning the rows and columns so that you can see if any particular region needs a certain number by the process of elimination. You repeat the process for every row and column. To make things even faster, scan the numbers in their order of frequency. Perform cross-hatching systematically by checking for all the digits from 1 to 9 in order.

Counting is the process of counting from 1 to 9 in row, columns, and regions so that you can tell if there are any missing numbers. Counting speeds up your solving time since you any numbers that you discover by counting are essentially “free guesses” since they don’t take a lot of analysis to discover. If you are working harder puzzles the value of one single cell can often be determined by counting in reverse. Counting in reverse is done by scanning the region, the row, and the column for numbers that *can’t* be right to see which numbers are left that might work.

Once you become an advanced Sukoku solver you will learn to start to look for what are called “contingencies” while you are scanning. This means that you will narrow down the location of a number within a row, column, or region to two or three cells. When each of those cells fall into the same row, or column, of the region, then you can use them to eliminate other numbers by cross-hatching and counting.

Sudoku puzzles that are really challenging might require you to try multiple contingencies. There will be times when you have to recognize these contingencies in multiple directions while at times even intersecting your number selection. A puzzle will be classified as “easy” if you can solve it by the scanning method alone. Sudoku puzzles that are more challenging won’t be solved by scanning alone but will need multiple solving strategies.

CHAPTER 6: BEGINNING THE CHALLENGE

Below is an unsolved Sudoku puzzle. It consists of a 9 x 9 grid that has been subdivided into 9 smaller grids of 3 x 3 squares. Each puzzle has a logical and a unique solution. To solve the puzzle, each row, column, and box must contain each of the numbers 1 to 9. Throughout

this guide the entire puzzle will be referred to the “grid”, a small 3 x 3 grid as a “region”, and the square that contains the number as the “cell”.

				8	3	4		
3					4	8	2	1
7								
		9	4		1		8	3
4	6		5		7	1		
								7
1	2	5	3					
		7	2	4				9

Rows and columns are referred to with row number first, followed by the column number:

4,5 is row 4, column 5

2,8 is row 2, column 8

Boxes are numbered 1 – 9 in reading order: 123 456 789

Guessing

Try not to guess. Until you have progressed to the touch and diabolical puzzles, guessing is not only totally unnecessary, but will lead you up paths that can make the puzzle virtually unsolvable. Simple logic is all that is required for gentle and moderate puzzles. Most puzzles that are rated easy to hard will require some sort of analysis.

Starting the Game

To solve Sudoku puzzles you will need to use logic. You need to ask yourself questions like “if a 1 is in this cell, will it go in this column?” or “if a 9 is already in this row, can a 9 go in this cell?” To make a start, look at each of the regions in the grid below and see which cells are empty, at the same time checking that cell’s column and row for a missing number. In this example, look at region 9. There is no 8 in the region, but there is an 8 in column 7 and in column 8. The only place for an 8 is in column 9, and in this box the only cell available is in row 9. So put an 8 in that cell. Once you have done this you have solved your first number.

				8	3	4		
3					4	8	2	1
7								
		9	4		1		8	3
4	6		5		7	1		
								7
1	2	5	3					9
		7	2	4				8

Continuing to think about 8, there is no 8 in region 1, but you can see an 8 in rows 1 and 2. So, in region 1, an 8 can only go in row 3, but there are 2 cells available. Make a note of this by penciling in a small 8! in both cells. Later, when you have found the position of the 8 in regions 4 or 7, you will be able to disprove one of your 8's in region 1. The more methodical that you are about solving your first Sudoku puzzles the better you will become at understanding the logic behind how you solve them. Take time when glancing through regions so that you don't scan through and miss an obvious number that you can place in a cell. Missing one number can set you back on how fast you solve the puzzle.

				8	3	4		
3					4	8	2	1
7	8	8						
		9	4		1		8	3
4	6		5		7	1		
								7
1	2	5	3					9
		7	2	4				8

You have been looking at region 9. As you can see, there is a 2 in regions 7 and 8, but none in region 9. The 2's in row 8 and row 9 mean the only place for a 2 in region 9 appears to be in row 7, and as there is already a 2 in column 8, there is only one cell left in that region for a 2 to go. You can enter the 2 for region 9 at 7,7.

As stated earlier, the more time you take in learning which strategy works best for certain puzzles the faster you will catch on to the logic behind the puzzle. Once you enter the number 2 in region 8 you will be ready to eliminate other numbers from other regions. Sudoku is all about filling in cells one by one by the process of elimination.

				8	3	4		
3					4	8	2	1
7	8	8						
		9	4		1		8	3
4	6		5		7	1		
						2		7
1	2	5	3					9
		7	2	4				8

There is a similar situation with the 4's in regions 4 and 5, but here the outcome is not so definite. Together with the 4 in column 7 these 4's eliminate all the available squares in region 6 apart from two. Pencil a small 4 in these two cells. Later on, one or other of your pencil marks will be proved or disproved.

				8	3	4		
3					4	8	2	1
7	8	8						
		9	4		1		8	3
							4	4
4	6		5		7	1		
						2		7
1	2	5	3					9
		7	2	4				8

Having proved the 2 in region 9 earlier, check to see if this helps you to solve anything else. For example, the 2 in region 3 shows where the 2 should go in region 6; it can only go in column 9, where there are two available squares. As you have not yet proved the position of the 4, one of the cells may be either a 4 or a 2.

				8	3	4		
3					4	8	2	1
7	8	8						
		9	4		1		8	3
							4	24
4	6		5		7	1		2
						2		7
1	2	5	3					9
		7	2	4				8

It's time for you to solve a number on your own. Take a look at region 8 and see where the number 7 should go. Continue to solve the more obvious numbers. There will come a point when you will need to change your strategy. The following puzzling solving tips will provide you with some schemes to solve the complete Sudoku puzzle. Some solvers base their entire strategy on schemes that they use consistently to solve certain puzzles.

CHAPTER 7: CHANGE OF STRATEGY

Once you have completed the steps in the previous chapter you may have come to realize that you need to change your strategy at some point. Easy Sudoku puzzles can be solved as the grid above was solved, but once you move on to more difficult puzzles you will need to come up with a different plan to find the right solution.

Searching for the Lone Number

		3	5	678	4	1		
	7			2			5	
5	2			178			6	
		8	6	5	9	3		
	3		127	4	17		8	
1469	149	5	8	1	3	7	49	1469
	5			1678			3	
	6			9			1	
		9	4	1678	2	5		

No matter what level of puzzle you are attempting to solve there are a few strategies that will allow you to get to a solution more quickly. The key strategy is to look for the lone number. In the following example, all the options for region 5 have been penciled in. At first there appear to be three places for the number 1 to go, but look between the 8 and the 3. There is a lone number 1.

It was not otherwise obvious that the only cell for the number 1 was row 6, column 5, as there is no number 1 in the immediate vicinity. Checking the adjacent regions and relevant row and column would not provide an immediate answer either – but no other number can go in that region.

		3	5	678	4	1		
	7			2			5	
5	2			178			6	
		8	6	5	9	3		
	3		127	4	17		8	
1469	149	5	8	1	3	7	49	1469
	5			1678			3	
	6			9			1	
		9	4	1678	2	5		

While the example uses pencil marks to illustrate the rule, more experienced solvers are quite capable of doing this in their head. Remember that this principle is true for regions, rows, and columns: If there is only one place for a number to go, then it is true for that region, and also the row and column it is in. You can eliminate all the other pencilled 1's in the region, row, and column.

Twins

Why limit yourself to one when sometimes two can do the job? In Sudoku you can easily become blind to the obvious. You might look at a region and think that there is no way of proving a number because it could go in more than one cell, but there are times when the answer is staring you right in the face. Sometimes the more obvious ways to find a solution is by looking at the obvious. Some solvers start by taking a few minutes to understand where the "givens" in the puzzle are laid out before they start to take any sort of solving action. This gives them a good feel for how easy or hard the puzzle is going to be so that they can apply certain strategies to their solving technique.

Take the following Sudoku.

5	4			9			7	2
2	7	9			3	6		4
9		8	7		4			
1	9	4	8			7		
7				5		4		9
			4	7	9	2		1
4			6			3		
		2	9	3			4	7
3	1			4				6

It is an example of a “easy” puzzle. A good start as already been made in finding the obvious numbers, but having just solved the 9 in region 4 you might be thinking about solving the 9 in region 1. It seems impossible, with just a 9 in row 1 and another in column 2 that immediately affect region 1.

But look more carefully and you will see that the 9 in row 8 precludes any 9 in row 8 of region 7. In addition, the 9 in column 2 eliminates the cell to the right of the 4 in that region, leaving just the two cells above and below the 2 in region 7 available for the 9. You have found a twin.

Pencil in these 9’s. While you don’t know which of these two will end up as 9 in this region, what you do know is that the 9 has to be in column 3. Therefore, a 9 cannot

go in column 3 of region 1, leaving it the one available cell in column 1.

5	4			9			7	2
2	7				3	6		4
9		8	7		4			
1	9	4	8			7		
7				5		4		9
			4	7	9	2		1
4		9	6			3		
		2	9	3			4	7
3	1	9		4				6

Triplets

In the previous example, having the “twins” did just as well as a solved number in helping you to find your number. But if two unsolved cells can help you on your way, three “solved” numbers together certainly can. All you need is to understand the concept behind looking for triplets. Look at the next example.

		4	6					
						3	8	6
3				9	7			2
	1			8	9		7	
9								1
	5		3	7			2	
6			8	4				7
2	8	1	7					
7	7	7	7		5	2		

Take a look at the sequence 2-8-1 in row 8. It can help you solve the 7 in region 8. The 7's in columns 5 and 6 place the 7's in region 8 at either 8,4 or 9,4. It is the 7 in row 7 that will provide you with sufficient clues to make a choice. Because there can be no more 7's in row 7, the 2-8-1 in row 8 forces the 7 in region 7 to be in row 9. Although you don't know which cell it will be in, the unsolved trio will prove that no more 7's will go in row 9, putting the 7 in region 8 at row 8. A solved row or column of three cells in a region is good news. Try the same trick with the 3-8-6 in row 2 to see if this triplet helps to solve any more of the puzzle.

CHAPTER 8: ELIMINATE THE EXTRANEOUS

We have looked at the basic number finding strategies, but what if these are just not up to the job? Until now we have been causally penciling in possible numbers, but there are many puzzles that will require you to be totally methodical in order to seek out and eliminate extraneous numbers.

If you have come to a point where obvious clues have dried up, before moving into unknown territory and beginning bifurcation (more on that later), you should ensure that you have actually found all the numbers that you can. The first step towards achieving this is to pencil in *all* possible numbers in each square. It takes less time than you would think to rattle off “can 1 go”, “can 2 go”, “can 3 go” while checking for these numbers in the cell’s region, row, and column.

It never hurts to repeat the one basic tenet of the Sudoku puzzle: if something is true for one element then it has to be true for the other two associated elements. Let’s look back to something that we looked at earlier: twins. When you discovering the rule about “twins” the grid wasn’t so crowded as it is in section of the Sudoku grid below.

9	46	5
68	2	148
7	3	148
1	8	2
5	7	6
4	9	3
3	146	149
68	5	189
2	146	7

This time the twins are mixed with other numbers. It's not obvious, but the two 1's in the top region are twins. While you don't know which cell is correct, you do know that the 1 in that region will exclude any other 1's in column 3 right the way down to the bottom cell. Using the twins strategy eliminates two 1's in that column of the bottom region. Two 1's in one region helped to eliminate 1's in another remote region.

The more numbers that you can eliminate from a region the easier it will be to determine where these eliminated numbers go on the grid. Some cells have obvious number choices and this makes it easy for you to start solving the

puzzle based on scanning and placing the numbers in the right cell.

9	46	5
68	2	148
7	3	148
1	8	2
5	7	6
4	9	3
3	146	49
68	5	89
2	146	7

It's important to show you this, because while nothing is actually solved by this action, eliminating those 1's could make all the difference in proving a number. You will be looking for things to help you move on in these kinds of crowded conditions. In a tough or diabolical puzzle it might allow you to proceed through to a solution without guessing.

Now you should look for matching pairs or trios of numbers in each column, row, and region. You have seen

matching numbers before: two squares in the same row, column, or region which share a pair of numbers. You can see the concept in the following illustration.

18	2	5	7	68	18	1689	3	4
----	---	---	---	----	----	------	---	---

In this sample row from a grid at column 1 there is a 1 and an 8 and at column 6 there is also a 1 and an 8. This matching pair is telling you that *only* either 1 or 8 is definitely at one or other of these locations. If that is true then *neither of these numbers can be at any other location in that row*. So you can eliminate the 1 and 8 in any other cell of the row where they do not appear together.

18	2	5	7	8	18	69	3	4
----	---	---	---	---	----	----	---	---

As you can see, this immediately solves the cell at column 5. This rule can be applied to a row, column, or region. Don't hesitate to try to use this rule on any Sudoku puzzle that you attempt to solve. It may not always work but you want to get into the habit of applying a variety of

solving strategies to any puzzle that you put your pencil to.

Three Numbers Exclusively

The number sharing rule can be taken a stage further. Say that you have three cells in a row that share the numbers 3, 7, and 9, and only those numbers. They may look like 3 7, 3 9, 7 9, or 3 7, 3 9, 3 7 9, or even 3 7 9, 3 7 9, 3 7 9. In the same way as the pair example worked, you can eliminate all other occurrences of those numbers anywhere else on that row (or column or region). It will probably take a minute or so to get your head around this one, but like the pairs, where you were looking for two cells that held the same two numbers exclusively, here we are looking for three cells that contain three numbers exclusively.

Sometimes, the obvious simply needs to be stated, as in the case of two cells that contain 3 7 and 3 7 9. If the 3 and the 7 occur *only* in those two cells in a row, column, or region, then either the 3 or the 7 must be true in either one of the cells. So why is the 9 still in that cell with what is so obviously a matching pair? Once that 9 has been eliminated, the pair matches, and can now eliminate other

3's and 7's in the row, column, or region. You could say this was a "hidden" pair.

You may find such hidden pairs in rows, columns, or regions, but when you find one in a region, only when it has be converted to a true matching pair can you consider it as part of a row or column. Hidden trios work in exactly the same way, but are just more difficult to spot. Once you have assimilated the principle of two numbers sharing two cells exclusively or three numbers sharing three cells exclusively, you will be well on the way to solving the most difficult Sudoku puzzles.

Step up the Action

It's important, if you want to successfully solve Sudoku puzzles that you take the time to attempt both easy and difficult puzzles using the concepts that you learn here. But what do you when all of the other methods have failed? In a nutshell, what you have to do is pick a likely pair of options in an unsolved cell and attempt to solve the puzzle using one of them. The method is called "bifurcation" which simply means taking a fork.

Since many books about Sudoku have been published there has been an amazing amount of discussion on the

Internet about extending the more satisfying elimination methods to solve Sudoku puzzles. So much so that solvers have come up with schemes for most puzzles without resorting to guessing methods at all. At this point it should be emphasized that Sudoku puzzles that may resist the methods discussed so far represent only a very small minority of the puzzles that are presented in most magazines, newspapers, and puzzle books. These puzzles will be among the diabolical and possibly some tough puzzles at the end of a book. They are valid puzzles, and many advanced Sudoku solvers have devised logic schemes (and computer programs) for solving them.

However, the wonderful thing about Sudoku puzzles is that you don't have to be a genius or a computer programmer to solve even the most diabolical example.

If you are meticulous and patient, and have mastered the gentle and moderate puzzles, then you can solve each and every Sudoku puzzle.

Hundreds, if not thousands, of people who do Sudoku puzzles are just ordinary, intelligent people. All it takes is a bit of time to sit down and enjoy the process of solving a puzzle.

CHAPTER 9: WHEN EVERYTHING ELSE FAILS

So what do you do when everything else fails? You will have to rely on bifurcation and methodological analysis. Those are the technical terms for the process of picking a likely pair of numbers, choosing one, and seeing where the number you have chosen gets you. Because you can be confident that *one* of the numbers will eventually produce a route to the solution, it is simply a matter of carefully analysing the options and testing your choice. If your first choice doesn't work out then you take the alternative route.

This final strategy is reserved for the most difficult of the diabolical and, occasionally, tough puzzles – when all else fails.

Think of the Sudoku puzzle as a maze. Gentle and moderate puzzles have a simple path straight through to the exit. Tough and diabolical puzzles may have “dead” ends which force you to try different routes. A tough puzzle is usually a more torturous version of a moderate Sudoku, but it may have one of these dead ends to cope with. Diabolical puzzles will have at least one dead end, and maybe more paths that you could follow before finding the number that leads to a logical exit. The way

to navigate this maze can be found in classical mythology. The following story will help to illustrate this.

Ariadne's Thread

Ariadne was the daughter of King Minos of Crete, who conquered the Athenian nation. An unfortunate intimacy between Ariadne's mother and a bull resulted in the birth of the monster – half bull, half man – called the Minotaur. The Minotaur was banished to spend his days in the Labyrinth. King Minos, being something of a tyrant, called for tribute from Athens in the form of young men and women to be sacrificed to the Minotaur.

The young Athenian hero, Theseus, offered to accompany a group of the young unfortunates into the Labyrinth so that he could kill the Minotaur and save Athens from the cruel tribute. Ariadne fell in love with Theseus and, not wishing to see him lost in the labyrinth once he had dealt with her bovine half brother, she provided him with a means of escape. She gave him a silken thread. Theseus had simply to unwind it while he went through the labyrinth; should he come to a dead end he could rewind it to the point where he had made a choice of paths and continue his search using the alternative route. The scheme worked out beautifully. The minotaur was slain

and Theseus found his way back out of the labyrinth. And Ariadne? Well, she got her ball of string back and became the moral of this story.

The tale of Ariadne should illustrate that the method of solving Sudoku puzzles using a “string” to bring you back to an alternative route when you find a dead end will work each and every time.

CHAPTER 10: SOLVING A DIABOLICAL PUZZLE

In its structure there is no difference between a tough Sudoku and a diabolical puzzle. The grading is only increased because in a diabolical puzzle there are more places where clues can run out and more apparent dead ends. You can find diabolical puzzles in a variety of places including puzzle books and on the Internet. If you buy a puzzle book you will find difficult and diabolical Sudoku puzzles after the easy to moderate puzzles. Always keep in mind that diabolical puzzles will take a bit more time to solve than easy to moderate puzzles. In fact, as you start to learn how to solve Sudoku puzzles it may take you over an hour to solve a difficult puzzle!

	8	4	5	69	1		7	2
			4		8	1		5
1	5			3	2	4	9	8
		9	8			5		1
8		1		5				9
4	6	5	9	1	3	2	8	7
7	4	3	1	8	5	9	2	6
5	9			4	6		1	3
	1		3		9		5	4

In the above example, ignore the pencil marks on the grid except for the first pair: at 1,5 you have either a 6 or a 9. There is at least one other pair that you could have chosen on the grid, but this was the first, so let's be logical and use that. If you choose to try the 6 first, the following grid shows the numbers that you will be able to complete using this number.

Pencil the now obvious numbers into the cells so that you bring the puzzle closer to a solution. You can always take a back step and find your back to where you started if you hit a dead end. Solving Sudoku puzzles is all about trying to follow the maze through to the end even though there may be many road blocks along the way.

9	8	4	5	6	1	3	7	2
2	3	7	4	9	8	1	6	5
1	5	6	7	3	2	4	9	8
3	7	9	8	2	4	5	X	1
8	2	1	X	5	7	6	3	9
4	6	5	9	1	3	2	8	7
7	4	3	1	8	5	9	2	6
5	9	8	2	4	6	7	1	3
6	1	2	3	7	9	8	5	4

But with just two cells to fill, look at what we have: at 4,8 the region needs a 4 to complete it, but there is already a 4 in that row at 4,6. Similarly, at 5,4 that region needs a 6, but one already exists in that row at 5,7. No second guess was needed to prove that at 1,5 the 6 was incorrect.

So now you need to return to 1,5 and try the 9. Now you are able to prove the 9 at 2,1 but nothing else is obvious; every cell is left with options. In this case you could leave both 9's because you have proved without doubt that the 6 at 1,5 could not be correct, but if the 6 had simply left you without sufficient clues, as the 9 did, you wouldn't

know which was true. Rather than start a new, uncertain path it is better to return to the situation you were in before you chose at 1,5 and find another cell to try from. This is a base that you know to be true.

	8	4	5	9	1		7	2
9			4		8	1		5
1	5			3	2	4	9	8
		9	8			5		1
8		1		5				9
4	6	5	9	1	3	2	8	7
7	4	3	1	8	5	9	2	6
5	9			4	6		1	3
	1		3		9		5	4

In the following grid you can see that you need to look at cell 1,7 , where the choice is between a 3 and a 6. When you choose the 3 you find yourself on a path that takes you to just two more to go but then you discover that a 2 is required to complete region 7. There is already a 2 in that row at 9,5. In region 9 you need an 8, but there is already an 8 in row 8 already. You now need to

wind the thread to get back to 1,7 where 3 was chosen last time. Trying 6 here will lead you to the solution.

6	8	4	5	9	1	3	7	2
9	3	2	4	7	8	1	6	5
1	5	7	6	3	2	4	9	8
3	2	9	8	6	7	5	4	1
8	7	1	2	5	4	6	3	9
4	6	5	9	1	3	2	8	7
7	4	3	1	8	5	9	2	6
5	9	8	7	4	6	X	1	3
X	1	6	3	2	9	7	5	4

CHAPTER 11: SAMPLE SUDOKU PUZZLES

Following are samples of Sudoku puzzles to challenge you no matter what level you are at. Easy puzzles are followed by more difficult ones. Solutions follow in the next chapter. Take your time as you learn to solve these puzzles, keeping in mind that you put the strategies learned in this guide to good use.

Puzzle 1:

1							5	
		6	3	8	2	9		
	4		1			2		
					7		9	
		4				5		
	5		9					
		1			3		7	
		3	6	9	1	8		
	8							9

Puzzle 2:

		9			3			7
	8		4			5		
4			5					
9	3		6			7		
2			1		7			6
		5			9		1	2
					2			8
		4			1		7	
1			3			6		

Puzzle 3:

7					1	3		
3							5	2
	4	1						
			6	4				5
	8						4	
9				3	7			
						2	9	
4	6							8
		8	5					1

Puzzle 4:

		6		4				
	4		2			1	5	
	8		1					2
						3	9	
2								5
	6	8						
4					6		3	
	5	7			4		8	
				5		2		

Puzzle 5:

	6	3			1	7		
4				8				
2					9			1
						6		9
	9						2	
3		1						
6			2					7
				6				8
		8	5			4	1	

Puzzle 6:

1					8	4	2	
			3					5
			2					9
	6	5			1			3
9			6			2	8	
3					4			
6					9			
	2	8	5					6

Puzzle 7:

		8				4		
	4		1		2		3	
6				3				8
7				4				1
	8		3		5		6	
3				6				5
1				7				3
	6		2		8		7	
		5				1		

Puzzle 8:

	4			1			9	
		1			8	6		
2			5					3
3				8			5	
		2				7		
	7			2				9
5					3			2
		6	7			4		
	9			4			8	

Puzzle 9:

		7			2			
	8			6			1	
3			5			9		
		2			5			7
	6						8	
5			4			3		
		8			6			9
	3			4			6	
			3			4		

Puzzle 10:

4							7	
8		1				2		
			2	3			9	
3			1		8	7		
7								1
		8	7		6			5
	5			8	1			
		7				6		9
	4							2

CHAPTER 12: SOLUTIONS

Puzzle 1:

1	3	2	4	6	9	7	5	8
5	7	6	3	8	2	9	1	4
8	4	9	1	7	5	2	6	3
2	6	8	5	4	7	3	9	1
9	1	4	2	3	6	5	8	7
3	5	7	9	1	8	4	2	6
4	9	1	8	5	3	6	7	2
7	2	3	6	9	1	8	4	5
6	8	5	7	2	4	1	3	9

Puzzle 2:

5	6	9	2	1	3	8	4	7
7	8	3	4	9	6	5	2	1
4	1	2	5	7	8	3	6	9
9	3	1	6	2	4	7	8	5
2	4	8	1	5	7	9	3	6
6	7	5	8	3	9	4	1	2
3	9	6	7	4	2	1	5	8
8	5	4	9	6	1	2	7	3
1	2	7	3	8	5	6	9	4

Puzzle 3:

7	2	5	9	6	1	3	8	4
3	9	6	8	7	4	1	5	2
8	4	1	3	2	5	6	7	9
1	3	7	6	4	8	9	2	5
6	8	2	1	5	9	7	4	3
9	5	4	2	3	7	8	1	6
5	1	3	4	8	6	2	9	7
4	6	9	7	1	2	5	3	8
2	7	8	5	9	3	4	6	1

Puzzle 4:

1	2	6	9	4	5	8	7	3
7	4	3	2	6	8	1	5	9
9	8	5	1	3	7	4	6	2
5	1	4	6	7	2	3	9	8
2	7	9	4	8	3	6	1	5
3	6	8	5	9	1	7	2	4
4	9	2	8	1	6	5	3	7
6	5	7	3	2	4	9	8	1
8	3	1	7	5	9	2	4	6

Puzzle 5:

9	6	3	4	5	1	7	8	2
4	1	7	3	8	2	9	6	5
2	8	5	6	7	9	3	4	1
8	4	2	1	3	5	6	7	9
5	9	6	8	4	7	1	2	3
3	7	1	9	2	6	8	5	4
6	3	4	2	1	8	5	9	7
1	5	9	7	6	4	2	3	8
7	2	8	5	9	3	4	1	6

Puzzle 6:

1	3	6	9	5	8	4	2	7
7	9	2	3	4	6	8	1	5
5	8	4	2	1	7	6	3	9
2	6	5	4	8	1	9	7	3
8	4	3	7	9	2	5	6	1
9	7	1	6	3	5	2	8	4
3	1	9	8	6	4	7	5	2
6	5	7	1	2	9	3	4	8
4	2	8	5	7	3	1	9	6

Puzzle 7:

2	3	8	9	5	6	4	1	7
5	4	7	1	8	2	9	3	6
6	1	9	4	3	7	2	5	8
7	5	6	8	4	9	3	2	1
9	8	1	3	2	5	7	6	4
3	2	4	7	6	1	8	9	5
1	9	2	5	7	4	6	8	3
4	6	3	2	1	8	5	7	9
8	7	5	6	9	3	1	4	2

Puzzle 8:

6	4	5	3	1	2	8	9	7
9	3	1	4	7	8	6	2	5
2	8	7	5	9	6	1	4	3
3	6	9	1	8	7	2	5	4
1	5	2	9	3	4	7	6	8
4	7	8	6	2	5	3	1	9
5	1	4	8	6	3	9	7	2
8	2	6	7	5	9	4	3	1
7	9	3	2	4	1	5	8	6

Puzzle 9:

1	5	7	8	9	2	6	4	3
9	8	4	7	6	3	2	1	5
3	2	6	5	1	4	9	7	8
8	4	2	6	3	5	1	9	7
7	6	3	1	2	9	5	8	4
5	9	1	4	7	8	3	2	6
4	1	8	2	5	6	7	3	9
2	3	5	9	4	7	8	6	1
6	7	9	3	8	1	4	5	2

Puzzle 10:

4	3	2	8	1	9	5	7	6
8	9	1	5	6	7	2	4	3
5	7	6	2	3	4	1	9	8
3	2	5	1	9	8	7	6	4
7	6		3	5	2	9	8	1
9	1	8	7	4	6	3	2	5
2	5	9	6	8	1	4	3	7
1	8	7	4	2	3	6	5	9
6	4	3	9	7	5	8	1	2

CONCLUSION

Once you learn the basics of solving a Sudoku puzzle you will need to apply it to puzzles of all levels of ability. You may find yourself struggling at first but this is just subjective and over time you will find yourself flying through puzzles that are mild to challenging. There are many mistakes that you will make along the way, and you may often find yourself wedged into a dead end with no where to go. Winding up the thread and starting again is the only way that you will learn to successfully solve a Sudoku puzzle.

The same puzzle may take one person thirty minutes to solve while another person may take two hours. Sometimes solving time comes down to your experience and other times it may just depend on how well you work the numbers. As well, there will be some days where you just have an innate ability to spot the clues. With practice, you will learn to spot these clues without even thinking about it.

No matter how good or bad you are solving Sudoku puzzles, you are guaranteed that you will get a good mental workout. As keep fit for the brain, Sudoku is as good as it gets.

Following is one last sample Sudoku puzzle so that you can put all the strategies and techniques that you learn in this guide to good use. Keep in mind that this Sudoku puzzle is rated “difficult” and therefore may take a bit of time for you to find the right solution.

DIABOLICAL!

		9	5		2			8
		5		8			9	
8		7			6			
6	1						7	
				9				
	9						8	3
			3			1		7
	4			5		8		
3			2		8	7		

DIABOLICAL SOLUTION!

1	3	9	5	7	2	4	6	8
4	6	5	1	8	3	2	9	7
8	2	7	9	4	6	3	1	5
6	1	2	8	3	5	9	7	4
5	8	3	4	9	7	6	2	1
7	9	4	6	2	1	5	8	3
9	7	8	3	6	4	1	5	2
2	4	1	7	5	8	8	3	6
3	5	6	2	1	8	7	4	9

CONGRATULATIONS AND HAPPY SOLVING!